Contents

Introduction 1

Chapter 1. Differential Equations and their Solutions 3
1.1. First Order ODE: Existence and Uniqueness 3
1.2. Euler's Method 16
1.3. Stationary Points and Closed Orbits 19
1.4. Continuity with Respect to Initial Conditions 22
1.5. Chaos—Or a Butterfly Spoils Laplace's Dream 25
1.6. Analytic ODE and their Solutions 32
1.7. Invariance Properties of Flows 34

Chapter 2. Linear Differential Equations 37
2.1. First Order Linear ODE 37
2.2. Non-autonomous First Order Linear ODE 48
2.3. Coupled and Uncoupled Harmonic Operators 50
2.4. Inhomogeneous Linear Differential Equations 52
2.5. Asymptotic Stability of Nonlinear ODE 53
2.6. Forced Harmonic Oscillators 55
2.7. Exponential Growth and Ecological Models 56

Chapter 3. 2nd Order ODE and The Calculus of Variations 63
3.1. Tangent Vectors and the Tangent Bundle 63
3.2. Second Order Differential Equations 66
3.3. The Calculus of Variations 68
3.4. The Euler-Lagrange Equations 70
3.5. Conservation Laws for Euler-Lagrange Equations 73
3.6. Two Classic Examples 74
3.7. Derivation of the Euler-Lagrange Equations 77
3.8. More General Variations 80
3.9. The Theorem of E. Noether 80
3.10. Lagrangians Defining the Same Functional 81
3.11. Riemannian Metrics and Geodesics 85
3.11. A Preview of Classical Mechanics 86

Chapter 4. Newtonian Mechanics 91
4.1. Introduction 91
4.2. Newton's Law's of Motion 93
4.3. Newtonian Kinematics 96
4.4. Classical Mechanics as a Physical Theory 99
4.5. Potential Functions and Conservation of Energy 106
4.6. One-dimensional Systems 111
4.7. The Third Law and Conservation Principles 118
4.8. Synthesis and Analysis of Mechanical Systems 122
4.9. Linear Systems and Harmonic Oscillators 125
4.10. Small Oscillations About Equilibrium 126

Chapter 5. Numerical Methods 133
5.1. Introduction 133
5.2. Fundamental Examples and Their Behavior 141
5.3. Summary of Method Behavior on Model Problems 167
5.4. Pairing for Error, Step-size, and Order Control 173

5.5 Behavior of Methods on a Model 2×2 System 176
5.6. Stiff Systems and the Method of Lines 183
5.7. Eulers Method: Convergence Analysis 199

Appendix A. Review of Metric and Normed Spaces 211
A.1. Linear Algebra and Analysis 211
A.2. Inner-Product Spaces 213

Appendix B. The Magic of Iteration 219
B.1 The Banach Contraction Principle 219
B.2 Newton's Method 224
B.3 The Inverse Function Theorem 226
B.4 The Existence and Uniqueness Theorem for ODE 227

Appendix C. Vector Fields as Differential Operators 229

Appendix D. Coordinate Systems and Canonical Forms 233
D.1 Local Coordinates 233
D.2 Some Canonical Forms 236

Appendix E. Parametrized Curves and Arclength 241

Appendix F. Smoothness with Respect to Initial Conditions 243

Appendix G. Canonical Form for Self-Adjoint Operators 245
G.1 The Spectral Theorem 245

Appendix H. Runge-Kutta Methods 249

Appendix I. Multistep Methods 267

Appendix J. Iterative Interpolation and its Error 291

Bibliography 294

Index 295