Algorithm F Test Case

As you probably realize by now, in order to have confidence in the correctness of computer programs, it is important to test them carefully—and in order to test them, one needs inputs for which the correct output is known. Since it may not be not entirely obvious how to construct a good test case for Algorithm F, let me suggest one possibility.

Recall that the spherical polar coordinates of a point x in $\mathbf{R}^3$ with cartesian coordinates (x_1, x_2, x_3) are defined by $r := \|x\|$, $\phi := \tan^{-1}(x_2/x_1)$ $\theta := \cos^{-1}(x_3/r)$. In the other direction, $x_1 = r\sin(\theta)\cos(\phi)$, $x_2 = r\sin(\theta)\sin(\phi)$, and $z := r\cos(\theta)$.

Let's use t_1 to denote the colatitude θ and t_2 to denote the longitude ϕ. Then we get a parametrization of the sphere through a point x and centered at the origin, by $(t_1, t_2) \mapsto \|x\| (\sin(t_1)\cos(t_2), \sin(t_1)\sin(t_2), \cos(t_1))$, with $0 \le t_1 \le \pi$, and $0 \le t_2 \le 2\pi$.

If we now differentiate with respect to t_1 and t_2, we find that these parametrizations of the family of spheres centered at the origin are solutions of the first order system of PDE:

$$\frac{\partial x}{\partial t_1} = X^1(x, t_1, t_2),$$
$$\frac{\partial x}{\partial t_2} = X^2(x, t_1, t_2),$$

where X^1 and X^2 are the maps $\mathbf{R}^3 \times \mathbf{R}^2 \to \mathbf{R}^3$ given by:

$$X^1(x, t_1, t_2) := \|x\| (\cos(t_1)\cos(t_2), \cos(t_1)\sin(t_2), -\sin(t_1)),$$
$$X^2(x, t_1, t_2) := \|x\| (-\sin(t_1)\sin(t_2), \sin(t_1)\cos(t_2), 0).$$

When you have finished defining AlgorithmF and want to test it, try it with this choice of X1 and X2, and use (0,0,r) as an initial condition at time t1 = t2 = 0. If you display the solution x using meshgrid, you should see a sphere of radius r displayed with latitude and longitude gridlines.